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Dynamic Pricing for Revenue Management in Retailing Using Support Vector Machine, Poisson Regression and Nonlinear Programming

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ABSTRACT

In recent years, dynamic pricing studies which depend on price-based revenue management have increased significantly due to the devolopments in predictive modeling softwares. Accordingly, studies dealing with the prediction of demand functions and price optimizations have also increased. In this research, a new methodology which could be used in retailing is suggested. In this context, support vector machine which depends on statistical learning and poisson regression which deals with count data is used separately in a comparative manner. In the result of comparisons, using the demand functions of the better forecasting model which has the lowest forecasting errors among them, price based revenue functions are generated. After this, in the case of unlimited capacity, taking the derivative of these previously obtained price based revenue functions or alternatively by using unconstrained nonlinear programming, optimal sales prices which maximized the relevant revenue functions are determined. In the case of limited capacity, price based revenue functions are rearranged according to the relation between price and demand and these rearranged revenue functions are proposed to be the objective function of nonlinear programming model given in this study. Adding capacity constraints to the model, similarly, optimal dynamic price policy which maximized revenue function of the retailer are constructed for the limited capacity conditions.

Keywords: Revenue Management, Dynamic Pricing, Support Vector Machine, Nonlinear Programming, Optimization, Poisson Regression.

JEL-Classification: C02, C45, C61

Destek Vektör Makinesi, Poisson Regresyonu ve Doğrusal Olmayan Programlama ile Perakendecilikte Gelir Yönetimi için Dinamik Fiyatlandırma

ÖZET

Son yıllarda, tahmine dayalı yazılımların gelişmesi ile birlikte, fiyat tabanlı gelir yönetimi ile ilgili dinamik fiyatlandırma çalışmaları giderek yaygınlaşmaktadır. Bu gelişmelerin bir yansıması olarak talep fonksiyonlarının tahmini ve fiyat optimizasyonu çalışmaları da önem kazanmıştır. Bu çalışmada, perakendecilik sektöründe kullanılabilecek yeni bir metodoloji önerilmiştir. Bu bağlamda, istatistiksel öğrenmeye dayanan destek vektör makinesi ve sayma verisi için önerilen poisson regresyonu karşılaştırmalı olarak kullanılmıştır. Karşılaştırma sonucunda, talebi daha düşük hata ile tahmin eden talep fonksiyonları kullanılarak, fiyata dayalı gelir fonksiyonları elde edilmiştir. Ardından, kapasitenin sınırsız olması durumunda, kısıtsız doğrusal olmayan programlama ile gelir fonksiyonlarını en büyükleyen optimal fiyat noktaları bulunmuştur. Kısıtlı kapasite ösz konusu ise, fiyata dayalı gelir fonksiyonları talep-fiyat ilişkisine göre talebe göre ifade edilmiş ve doğrusal olmayan programlama modelinin amaç fonksiyonunu oluşturmuştur. Modele mevcut kapasite kısıtları da eklenerek, benzer şekilde, kısıtlı kapasite için perakendecinin gelir fonksiyonunu en büyükleyen optimal dinamik fiyat politikası oluşturulmuştur.

Anahtar Kelimeler: Gelir Yönetimi, Dinamik Fiyatlandırma, Destek Vektör Makinesi, Doğrusal Olmayan Programlama, Optimizasyon, Poisson Regresyon.

1. INTRODUCTION

When a revenue management application is executed, it must be ensured that optimal sales prices which maximize revenue should be obtained. So, “Revenue Management” concept which has emerged in recent years has been dealing with finding out optimal prices in order to maximize revenue.

Changing price throughout managing revenue is called as “ dynamic pricing”. In this context, it requires difficult and comprehensive analysis to decide how often and how much the price must be changed. In general, the problem is defined as finding out optimal dynamic prices to maximize expected revenue in a limited planning horizon with a fixed amount of inventory on hand.(Monohan et al., 2002)

By the way, dynamic pricing applications have been performing well when the capacity is immediately perishable, sale period is short and the relevant demand is price sensitive. All these properties can be seen in retailers dealing with marketing seasonal goods.

Looking at revenue management applications, most of the studies constitute the pricing of airline flight tickets and the pricing of hotel rooms. Whereas; the studies in retailing in literature are much less with respect to other industries.

In this study different from previous studies, demand functions obtained by support vector machine and poisson regression have been compared for prediction accuracy. Since the data used in the study have the property of count data, the efficiency of support vector machine and poisson regression for such count data has been investigated as another aim. In the result of analysis, using the demand functions which have belonged to the better forecasting model, price based revenue functions have been obtained. After this, in the case of unlimited capacity, taking the derivative of these previously obtained price based revenue functions or alternatively using unconstrained nonlinear programming, optimal sales prices have been gathered which have maximized the relevant revenue functions.

On the other hand, when the capacity constraints are included in the problem, the previously obtained price based revenue functions are reformulated according to the demand. This reformulation has been performed using the relation between price and demand. Then, these reformulated revenue functions have been appointed as the objective function of the nonlinear programming model and capacity constraints have been taken into consideration. So, in case of limited capacity, optimal dynamic sales maximizing the revenue have been discovered.

The remainder of this paper organized as follows. A literature review concerning the subject is provided in section 2. Procedures of the methodology and the models used in the methodology are explained in section 3. A real case study with numerical results generated using the proposed method and comparisons are given in section 4. Finally, the concluding remarks are presented in section 5.

2. LITERATURE REVIEW

The most significant studies dealing with revenue management and dynamic pricing in retailing literature are summarized as follows.

Badinelli and Olsen (1990) illustrated that when it was required to make a price decison, it should not be forgotton that today's price would affect the future price.

Gallego and Van Ryzin (1994) suggested that fixed price policy was optimal as the sales volume tended to be infinite.

Bitran and Wadhwa (1996) defined the seasonal good pricing problem as finding out the dynamic optimal pricing policy of a retail good with a fixed amount of inventory and limited sales horizon. In the relevant study, authors suggested some assumptions about the structure of the problem and analyzed it according to these assumptions.

Chatwin (2000) studied a problem in which there were limited number of goods to be sold before a certain time point and the retailer had to price in order to get the maximum revenue. In his study, the retailer also had to make the price decions through a given allowed price options.

Lippman (2003) explained that retailing revenue management applications were able to offer optimal price solutions which change in the course of time with respect to a retailer's goals.

Elmaghraby and Keskinocak (2003) offered a literature survey and certain studies about dynamic pricing.

Gabriel Bitran and Rene Caltendey (2003) investigated a problem in which customers were price sensitive and the retailer had to sell its inventory before a deadline. In the relevant study, the aim of the retailer is to find an optimal price strategy which would sell its inventory with the maximum revenue.

Hawtin (2003) clarified that the business rules should be taken into consideration and managed well when using revenue management principles in retailing. In his study, it has been suggested that the prices obtained from a revenue management should not be in conflict with the company's image. Otherwise, it would result in not only losing potential sales but also losing customers forever.

Lin (2004) studied a problem in which a seller had a limited inventory on-hand and had to decide the optimal price strategy to obtain maximum revenue. In the context of the problem, some assumptions about the customer demand were made and the problem was analyzed according to these assumptions.

Ziya, Ayhan and Foley (2004) analyzed the three most well-known assumptions in the revenue management literature. These assumptions were decreasing marginal revenue with respect to demand, decreasing marginal revenue with respect to price and increasing price sensitivity of demand.

Kuyumcu (2007) explained that revenue management dealt with too many disciplines such as operations research, mathematics, statistics, marketing and finance. By the way, it was indicated that the best way to increase a firm's revenue could be achieved by using right prices at the right time. For this goal, the accuracy of demand prediction models should be analyzed. For example, it was illustrared in the study that demand functions depending on regression should be analyzed by statistical performance measures such as correlation coefficient.

Shields and Shelleman (2009) emphasized that revenue management applications were made up of four basic parts. These were defined as obtaining customer demand data, analyzing demand data, balancing demand and supply and deciding optimal prices to maximize revenue. By the way, a control list which would guide managers were suggested.

Farias and Van Roy (2010), studied a problem in which a seller had a limited given number of inventory on-hand and had to decide the best price strategy to obtain maximum revenue. In the context of the problem, the authors did not utilize the past demand data. Instead of this, according to assumptions on customer arrivals and customer buyer behavior, optimal prices were tried to be found out.

Zhao, Tian and Li (2012) studied a dynamic pricing problem based on consumer behavior. In the scope of the problem, they tried to figure out how consumption inactivity and slowdown in consumer buying behavior affected on dynamic pricing strategy and optimal prices. The results of the study proposed the slowdown in consumer buying behavior has negatively affected on expected revenues of the business and optimal dynamic pricing policy.

Zhou and Li (2014) focused the consumer strategy behavior effect on retailers pricing mechanism. Under the condition of uncertainty demand and deterministic demand, they found that strategy behavior of consumers influenced price and profit. By introducing a discount factor, considering inventory timely complement and fixed inventory in two cases, they obtained purchase decision and dynamic pricing startegies of consumers.

Levin, Nediak and Bazhanov (2014) considered a dynamic pricing problem for a monopolistic company selling a perishable product when customer demand has been both uncertain and occured in batches that must be fulfilled as a whole. The seller can price-discriminate between different sized batches by setting different unit prices. The problem was modeled as a stochastic optimal control problem to find an inventory-contingent dynamic pricing policy that maximized expected total revenues. They found the properties of optimal pricing policy and proved several monotonicity results.

3. METHODOLOGY

As mentioned above the methodology proposed in this study has two parts. The first part is to forecast demand functions and the second part is to generate the price based revenue functions. The second part is investigated under the two different conditions such as unlimited and limited capacity to reach optimal sales prices and dynamic sales prices respectively for maximizing the revenue functions. Function blocks of the proposed methodology is given in Figure 1.

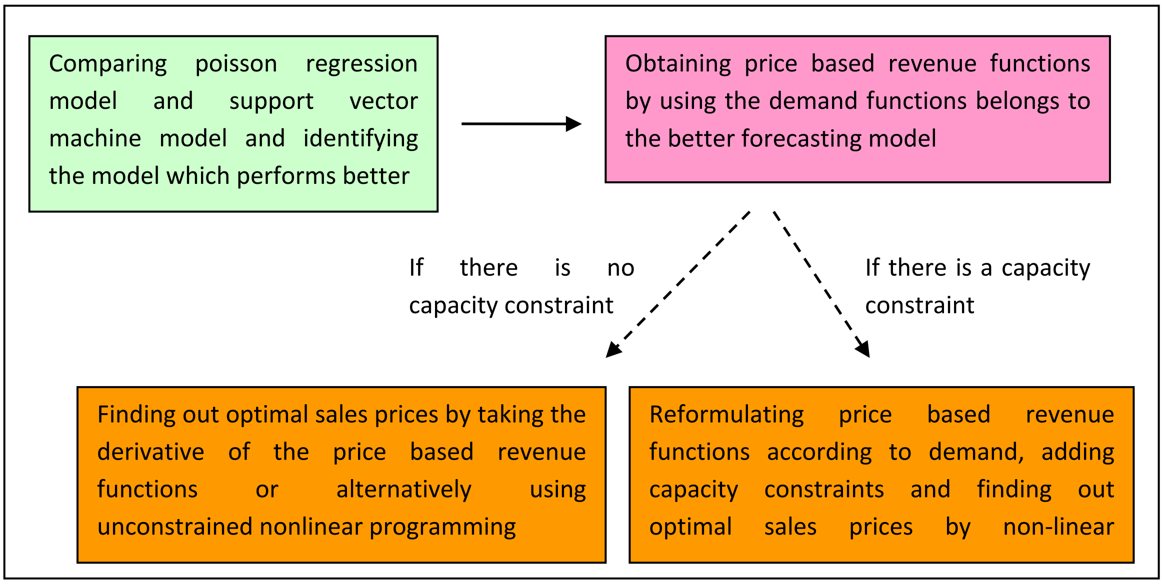


Figure 1: Function blocks of the suggested methodolgy

In the following subtitles, detailed information has been given about the models of the methodology used in the context of this study.

3.1 Support Vector Machine

Support vector machine is a supervised learning procedure which analyze data and learn from data. It is used in classifying and regression analysis. Support vector machine was suggested firstly by Vladimir Vapnik and his co-workers in 1992 at Computational Learning Theory Conference. It has reached its last shape which is used now in 1995 by Corinna Cortes and Vladimir Vapnik. In 1997, support vector machine algorithm were expanded to include the regression applications by again Vapnik and his colleagues. Since then, it has been using increasingly by the improvements in computer technology in the last decade.

Although the first studies in support vector machine were about classifying, in latter studies, very satisfying results were obtained about predicting time series and regression applications. General information about regression application of support vector regression is as follows.

For a learning set like $D = \{(x_1, y_1), (x_2, y_2), \ldots, (x_l, y_l)\}$

Here, x_i is the value of an input variable inside N-dimensional input variables and y_i is the value of an output variable;

$f(x) = \langle w, x \rangle + b$ function is tried to be found out. Here, w is called as “normal vector” and x is called as “input vector”. This is a vectoral multiplication of two vectors with the equal dimensions.

The basic goal in support vector regression is to find such a function of f(x) that the real y_i output values are predicted with at most ε deviation. If it is not the case to find such an f function with ε error for all data points, then a constant is used to determine the trade off between the flatness of f and the amount up to which deviations larger than ε are tolerated. So, only the deviations larger than ε are penalized. Figure 2 depicts the situation graphically.

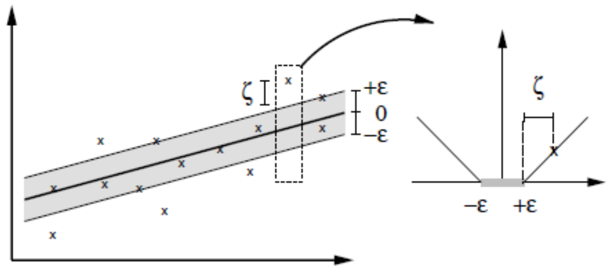


Figure 2: Situation of predicting with larger than ε error
(Smola and Schölkopf, 2004)

As it can be seen in Figure 3 below, the f(x) function should predict the real ouput data with ε error or less.

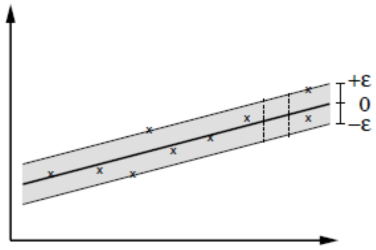


Figure 3: Predicting real output data with $\pm\varepsilon$ error
(Smola and Schölkopf, 2004)

In order to find such a function of f(x); a minimum w vector is looked for. So, the norm of w vector is minimized. The problem can be written as convex optimization as follows:

$$\min \quad \frac{1}{2}\|w\|^2 \tag{1}$$

$$\text{Constraints}: \; y_i - \langle w, x_i \rangle - b \le \varepsilon \tag{2}$$

$$\langle w, x_i \rangle + b - y_i \le \varepsilon \tag{3}$$

However, a feasible solution with ε error in the extent of the above constraints might not be found. To overcome this problem, ξ_i and ξ_i^* free variables have been added to the model. (Smola and Schölkopf, 2004)

According to this, the problem is reformulated as follows:

$$\min \quad \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{l}(\xi_i + \xi_i^*) \tag{4}$$

Constraints : $y_i - \langle w, x_i \rangle - b \leq \varepsilon + \xi_i$ (5)

$$\langle w, x_i \rangle + b - y_i \leq \varepsilon + \xi_i^* \tag{6}$$

$$\xi_i,\ \xi_i^* \quad \geq 0 \tag{7}$$

Let C is a constant coefficient which is larger than 0 and helps to compromise with deviations larger than ε and the model complexity. The reason to use support vector machine is to specify how this model performs in the prediction of demand and obtain the relevant demand function.

3.2 Poisson Regression

Poisson regression model is also known as log-lineer regression. Poisson regression depends on exponential distribution. By the way, it has been widely used since it does not require the normal distribution assumptions. (Frome et al., 1973)

Poisson regression analysis explain the relation between independent variables and a dependent variable which is based on count data. Count data is defined as the occuring number of a particular event in a certain time period. So, count data is a discrete number such as 0, 1, 2, or 3. For such data, side effects in a medical treatment or number of demand in a certain period are good examples. First studies in count data modelling were seen in actuarial sciences, biostatistics and demography. Then, it has taken much consideration in econometrics and has been especially used in microeconomics.

In poisson regression models, the relation function which relates the linear structure of independent variables to the dependent variable's expected value is executed by logarithm (Frome, 1983)

As Y denoting dependent variable, X_i denoting independent variables and C_i denoting coefficients of independent variables; relation function is as follows:

$$\log_e(Y) = C_0 + C_1 * X_1 + C_2 * X_2 + \ldots + C_N * X_N \tag{8}$$

So,

$$Y = e^{C_0 + C_1 * X_1 + C_2 * X_2 + \ldots + C_N * X_N} \tag{9}$$

Similarly,

$$Y = e^{C_0} * e^{C_1 * X_1} * e^{C_2 * X_2} * \ldots * e^{C_N * X_N} \tag{10}$$

The reason to use poisson regression is to specify how this model performs in the prediction of demand and obtain the relevant demand function.

The most significant property of poisson distribuition is that it assumes that the mean and variance are equal. If mean is less than variance, it is called as overdispersion. In such situations, it is suggested to use regression analysis which involves dispersion parameter in order to explain extra-poisson condition. The relevant regression analysis are negative binomial regression and mixed poisson regression analysis. In this study, overdispersion tests have been executed in SAS and since no overdispersion sitiuation has been encountered, poisson regression has been approved.

3.3 Non-linear Programming

Nonlinear programming is the process of solving a system of equalities and inequalities, collectively termed constraints, over a set of unknown real variables, along with an objective function to be maximized or minimized, where some of the constraints or the objective function are nonlinear.

For a maximization problem, if the objective function is concave and solution space bounded with constraints is convex, convex optimization models can be used. Similarly, for a minimization problem, if the objective function is convex and solution space bounded with constraints is convex, convex optimization models can be used.

In this study, nonlinear programming is used to analyze demand based revenue functions in constrained capacity conditions in order to find out optimal dynamic sales prices.

In the following, constants and variables used in the nonlinear programming model is defined.

T = Sales season
t = Periods of sales season (from 1 to T)
d_t = Demand corresponding to optimal price (p_t)
p_t = optimal price for each period
$r_t(d_t)$ = Revenue function based on demand at each period
$J_t(d_t)$ = Derivative of revenue function at each period (marginal revenue)
λ = Lagrange multiplier
C = On hand inventory at the beginning of the sales season

Regarding the constants and variables above, the goal function and constraints of nonlinear programming model is given below.

$Max\sum_{t=1}^{T} r_t(d_t)$ Sum of total revenues should be maximized and concave (11)

$\forall\ J_t(d_t) = \lambda$ Marginal revenues for each period should be equal to lagrange multiplier (12)

$\sum_{t=1}^{T} d_t \leq C$ Total demand may not exceed on hand inventory at the beginning of the season (13)

$\lambda(C - \sum_{t=1}^{T} d_t) = 0$ If there is excess inventory at the end of the season, lagrange multiplier should be equal to zero. This is called complementary slackness. (14)

$\forall d_t \geq 0$ Positivity constraint (15)

$\lambda \geq 0$ Lagrange constraint (16)

3.4. Mathematical Model for Suggested Methodology

Mathematical model for suggested methodology is as follows.

T = Sales season
i = Function index
t = Sales period
d_{ti} = demand function i at period t
p_{ti} = price for function i at period t
E_{ti} = Forecast error for function i at period t
AE_t = Advertising expense at period t
$d_{ti}(p_{ti}) = F_{ti}$ price based demand fuction for function i at period t
$R_{ti}(p_{ti})$ = price based revenue function for function i at period t
a,b,c,d,e = arbitrary constants
exp = 2,718 (euler number)

Unlimited capacity condition

1) Finding price based demand functions at each period

For $AE_t = e$, $d_{ti}(p_{ti}) = F_{ti}$ $(F_{t1} = a \times exp^{pt1+b}$, $F_{t2} = c \times p_{t2} + d)$ (17)

2) Finding price based revenue functions at each period

$R_{ti}(p_{ti}) = F_{ti} \times p_{ti}$ (18)

3) Obtaining price based revenue functions using the least error demand functions and finding optimal prices.

$$\text{Max } R_{ti}(p_{ti}) = \sum_{t=1}^{T} [F_{ti}(\min E_{ti}) \times p_{ti}] \quad (19)$$

From the equation above, optimal prices are computed.

Limited capacity condition

1) Price based revenue functions are defined based on demand

$d_{ti}(p_{ti})$ = price based demand function for function i at period t

$p_{ti}(d_{ti})$ = demand based price function for function i at period t

$p_{ti}(d_{ti}) = G_{ti}$ $(G_{t1} = 1/a\,[b - \ln(d_{t1})]$, $G_{t2} = (c - d_{t2})/d)$ (20)

$R_{ti}(d_{ti}) = G_{ti} \times d_{ti}$ (21)

$J_{ti}(d_{ti})$ = derivative of demand based revenue function i at period t

2) Defining price based revenue functions as the goal function using the least error demand functions

$$\text{Max } R_{ti}(d_{ti}) = \sum_{t=1}^{T} [G_{ti}(\min E_{ti}) \times d_{ti}] \quad (22)$$

3) Adding capacity constraints, complementary slackness and positivity constraints into nonlinear programming model

$$\sum_{t=1}^{T} d_{ti} \le C \quad (23)$$

$$\forall\, J_{ti}(d_{ti}) = \lambda \quad (24)$$

$$\lambda(C - \sum_{t=1}^{T} d_{ti}) = 0 \quad (25)$$

$$\forall\, d_{ti} \ge 0 \quad (26)$$

$$\lambda \ge 0 \quad (27)$$

4) Using optimal (d_{ti}) finding out optimal prices

$p_{ti}(d_{ti}) = G_{ti}$ (using inverse function, optimal prices are found)

5) Revising optimal price strategy if actual demand is different than forecasted demand when optimal price strategy is applied

Cj = on hand inventory at period j

T-j = Time to remain until the end of the sales season

$$\text{Max } R_{ti}(d_{ti}) = \sum_{t=j}^{T} [G_{ti}(\min E_{ti}) \times d_{ti}] \quad (28)$$

$$\sum_{t=j}^{T} d_{ti} \le C_j \quad (29)$$

$$\forall\, J_{ti}(d_{ti}) = \lambda \quad (30)$$

$$\lambda(C_j - \sum_{t=j}^{T} d_{ti}) = 0 \quad (31)$$

$$\forall\, d_{ti} \ge 0 \quad (32)$$

$$\lambda \ge 0 \quad (33)$$

$p_{ti}(d_{ti}) = G_{ti}$ (using inverse function, optimal prices are found)

6) Selling more than expected and replenishing inventory

ζ = replenishment amount at period j

$$\text{Max } R_{ti}(d_{ti}) = \sum_{t=j}^{T} [G_{ti}(\min E_{ti}) \times d_{ti}] \quad (34)$$

$$\sum_{t=j}^{T} d_{ti} \le C_j + \zeta \quad (35)$$

$$\forall\, J_{ti}(d_{ti}) = \lambda \quad (36)$$

$$\lambda(C_j + \zeta - \sum_{t=j}^{T} d_{ti}) = 0 \quad (37)$$

$$\forall\, d_{ti} \geq 0 \quad (38)$$

$$\lambda \geq 0 \quad (39)$$

$p_{ti}\,(d_{ti}) = G_{ti}$ (using inverse function, optimal prices are found)

4. APPLICATION AND ANALYSIS

In the scope of application, different type of two seasonal goods sold by a business have been taken into consideration and monthly past price, monthly advertising expense and monthly sales data have been gathered for each of the goods. By the way, the monthly advertising expense the firm will make the next year is already known. In order to make right and consistent calculations with the obtained monthly data belonging to the past years, all the data has been reformulated according to CPI (consumer price index) rates. So, monetary values have been purified from inflation effects. After this, it has been approved to predict demand using poisson regression and support vector machine.

For the comparison of the forecasting accuracy of support vector machine and poisson regression model, relative root mean squared error (RRMSE) of the models have been used. By the way, for the better forecasting model, it has also been investigated whether a forecasting bias exists or not. Using RRMSE, the model which predicts demand better has been identified among support vector machine and poisson regression model. Then, it has been demonstrated using tracking signal that the models predicting better do not have forecasting bias. So the demand functions of these better predicting models have been used in non-linear programming and optimal dynamic sales prices have been found out.

To obtain the demand functions for support vector machine, Weka 3.6.3 machine learning software has been used. Besides that, in order to introduce data to Weka software, csv (comma seperated value) format has been used. A critical issue in the computation process of support vector machine is that the normalization of the data should not be applied. This must be ensured in order to compare poisson regression and support vector machine in a significant and meaningful way. To obtain the demand functions for poisson regression, Eviews 7 software has been used. Similarly to introduce data to Eviews 7 software, csv (comma seperated value) format has been used.

Product 1									Product 2								
Period 1			Period 2			Period 3			Period 1			Period 2			Period 3		
deviance	scaled deviance	LL	deviance	scaled deviance	LL	deviance	scaled deviance	LL	deviance	scaled deviance	LL	deviance	scaled deviance	LL	Deviance	scaled deviance	LL
0.887	0.761	0.808	0.782	0.824	0.605	0.773	0.74	0.79	0.641	0.679	0.6	0.703	0.752	0.68	0.796	0.691	0.694

Table 1: Poisson regression overdispersion criteria obtained from SAS

As it can be seen from the above Table 1, since overdispersion criteria values are not larger than 1, there is no overdispersion being identified. If these values were larger than 1, then overdispersion would exist. Table 2 and Table 3 below provides RRMSE values for both support vector machine and poisson regression.

			RRMSE	Demand Function
Product 1	Period 1	Support Vector Machine	4,87%	-0,3894 x Price + 0,0737 x Advertising Expense - 82,3188
		Poisson Regression	**4,48%**	$e^{2,030909 - 0,015110 \text{ x Price} + 0,001393 \text{ x Advertising Expense}}$
	Period 2	Support Vector Machine	7,16%	-0,6665 x Price + 0,0286 x Advertising Expense + 28,6666
		Poisson Regression	**7,15%**	$e^{3,572218 - 0,023363 \text{ x Price} + 0,000908 \text{ x Advertising Expense}}$
	Period 3	Support Vector Machine	10,92%	-0,4049 x Price + 0,0757 x Advertising Expense - 100,3129
		Poisson Regression	**9,58%**	$e^{2,178101 - 0,031788 \text{ x Price} + 0,001752 \text{ x Advertising Expense}}$

Table 2: RRMSE values which belong to support vector machine and poisson regression for Product 1

			RRMSE	Demand Function
Product 2	Period 1	Support Vector Machine	4,56%	-0,2126 x Price + 0,0757 x Advertising Expense - 94,9189
		Poisson Regression	**3,75%**	$e^{0,916714 - 0,010304 \text{ x Price} + 0,001915 \text{ x Advertising Expense}}$
	Period 2	**Support Vector Machine**	**6,13%**	**-0,797 x Price + 0,0329 x Advertising Expense + 64,1159**
		Poisson Regression	7,18%	$e^{8,178779 - 0,039923 \text{ x Price} + 0,000183 \text{ x Advertising Expense}}$
	Period 3	**Support Vector Machine**	**11,92%**	**-0,5625 x Price + 0,0667 x Advertising Expense - 45,0879**
		Poisson Regression	13,32%	$e^{2,994172 - 0,05348 \text{ x Price} + 0,0034 \text{ x Advertising Expense}}$

Table 3: RRMSE values which belong to support vector machine and poisson regression for Product 2

As it can be seen from Table 2 and Table 3, according to RRMSE values for each product, the forecasting accuracy of the forecasting models indicated bold are better than others in the relevant period (month). Another issue different from forecasting accuracy is the forecasting bias. To measure the forecasting bias of a forecasting model, tracking signal is used. Tracking signal is found simply by dividing cumulative error to the mean absolute error. The tracking signal which measures forecasting bias actually computes how often a forecasting model forecasts over actual demand or under actual demand.

High positive tracking signals mean that the forecasting model is consistently inclined to forecast under actual demand which is also called as “under-forecasting bias”. On the other hand, high negative tracking signals mean that the forecasting model is consistently inclined to forecast over actual demand which is also called as “over-forecasting bias”. Thereby, high negative and positive tracking signal values indicate that the forecasting model is out of control such as in the quality control charts. So, in order to check whether a forecasting bias exists or not, ± 4 tracking signal value is used which corresponds to 3 standard deviation. As a result, forecasting accuracy and forecasting bias are different subjects and should be taken in hand seperately. For example, a forecasting model which is good at forecasting accuracy by RRMSE, may have poor results in terms of forecasting bias if it is evaluated by tracking signal.

In the scope of the explanations above, for each product the forecasting models which forecast demand better by RRMSE have been analyzed by tracking signal in terms of forecasting bias. In Figure 4 below, the tracking signal relevant to poisson regression for product 1 in period 1 is shown. In Figure 5 below, the tracking signal relevant to poisson regression for product 1 in period 2 is shown. In Figure 6 below, the tracking signal relevant to poisson regression for product 1 in period 3 is shown. In Figure 7 below, the tracking signal relevant to poisson regression for product 2 in period 1 is shown. In Figure 8 below, the tracking signal relevant to support vector machine for product 2 in period 2 is shown. In Figure 9 below, the tracking signal relevant to support vector machine for product 2 in period 3 is shown.

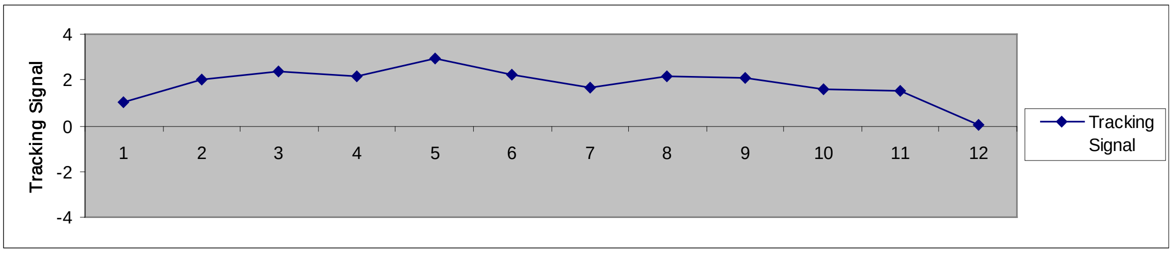


Figure 4: Tracking signal relevant to poisson regression for product 1 in period 1

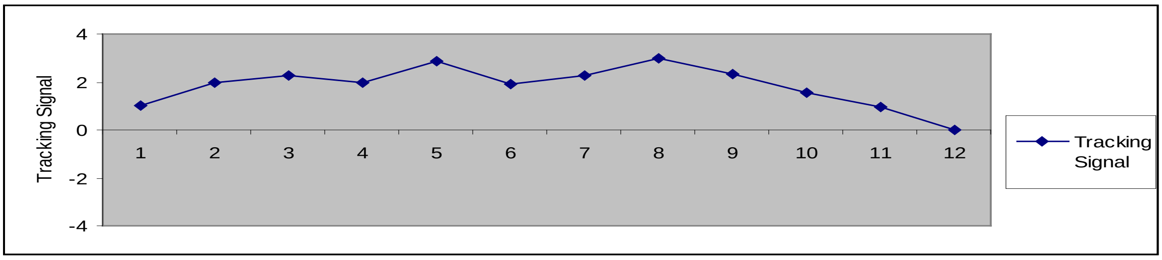


Figure 5: Tracking signal relevant to poisson regression for product 1 in period 2

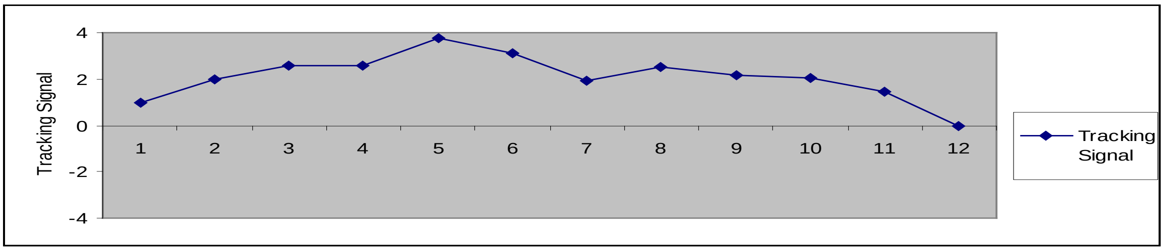


Figure 6: Tracking signal relevant to poisson regression for product 1 in period 3

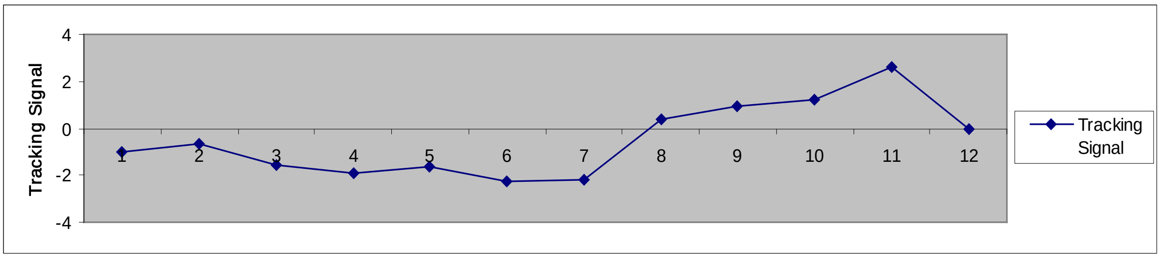


Figure 7: Tracking signal relevant to poisson regression for product 2 in period 1

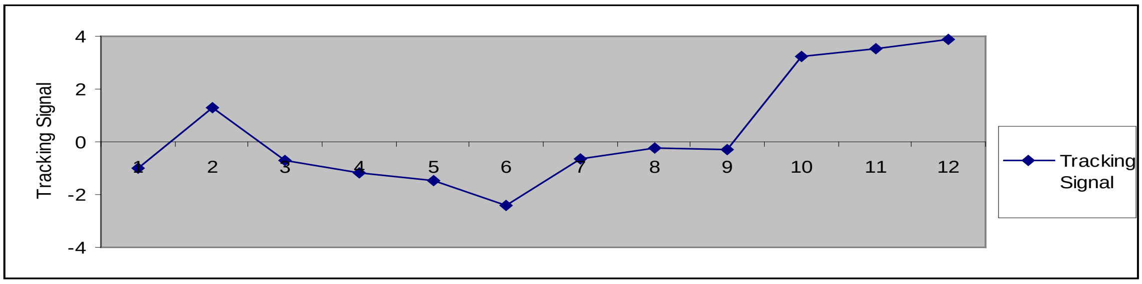


Figure 8: Tracking signal relevant to poisson regression for product 2 in period 1

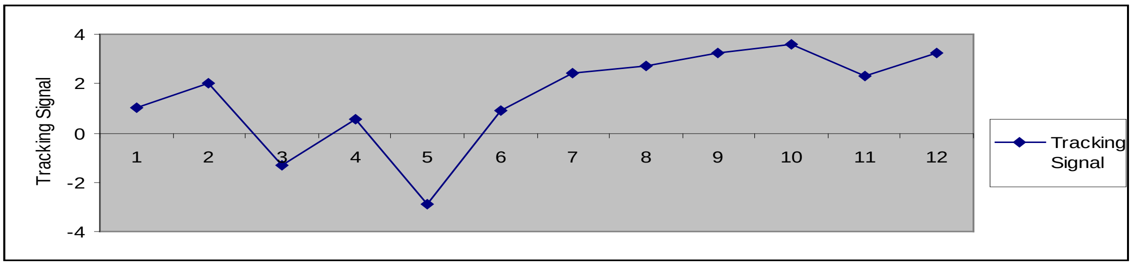


Figure 9: Tracking signal relevant to support vector machine for product 2 in period 3

As it can be seen from the figures above for both products, the forecasting models which forecast demand better are also convenient in terms of forecasting bias when evaluated by tracking signal since the ± 4 control limits are not violated. So, the demand functions of these forecasting models have been used in further analysis.

Since the montly advertising expense for the next year is 2000 Turkish Liras (TRY), the demand functions of the forecasting models which forecast demand better have been used and the following price based demand functions have been obtained.

Price Based Demand Functions For Product 1

$d_1 = e^{2,030909 - 0,015110 \text{ x Price} + 0,001393 \text{ x } 2000} = e^{4,816909 - 0,015110 \text{ x Price}}$ (40)

$d_2 = e^{3,572218 - 0,023363 \text{ x Price} + 0,000908 \text{ x } 2000} = e^{5,388218 - 0,023363 \text{ x Price}}$ (41)

$d_3 = e^{2,178101 - 0,031788 \text{ x Price} + 0,001752 \text{ x } 2000} = e^{5,682101 - 0,031788 \text{ x Price}}$ (42)

Price Based Demand Functions For Product 2

$d_1 = e^{0,916714 - 0,010304 \text{ x Price} + 0,001915 \text{ x } 2000} = e^{4,746714 - 0,010304 \text{ x Price}}$ (43)

d_2 = -0,797 x Price + 0,0329 x 2000 + 64,1159 = -0,797 x Price + 129,9159 (44)

d_3 = -0,5625 x Price + 0,0667 x 2000 – 45,0879 = - 0,5625 x Price + 88,3121 (45)

In the next step, multiplying the price based revenue functions in the above by price, the following price based revenue functions have been obtained.

Price Based Revenue Functions For Product 1

$e^{4,816909 - 0,015110 \text{ x Price}}$ x Price (46)

$e^{5,388218 - 0,023363 \text{ x Price}}$ x Price (47)

$e^{5,682101 - 0,031788 \text{ x Price}}$ x Price (48)

Price Based Revenue Functions For Product 2

$e^{4,746714 - 0,010304 \text{ x Price}}$ x Price (49)

- 0,797 x Price^2 + 129,9159 x Price (50)

- 0,5625 x Price^2 + 88,3121 x Price (51)

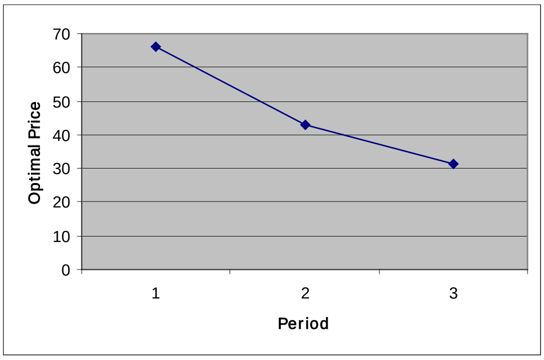
4.1 Computing The Optimal Dynamic Pricing Strategy for Unlimited Capacity Constraint Condition

In the above, nonlinear price based revenue functions have been obtained between (46) and (51). Since there is no capacity constraint assumed, optimal dynamic prices below in table 4 have been obtained by applying derivatives to these price based revenue functions. In table 4, the corresponding integer demand and revenue values have also been shown.

		Optimal Price	Demand	Revenue	Cumulative Revenue
Product 1	Period 1	66,18	45	2978	2978
	Period 2	42,80	80	3424	6402
	Period 3	31,50	107	3371	9773
Product 2	Period 1	97,04	42	4075	4075
	Period 2	81,50	65	5297	9372
	Period 3	78,50	44	3454	12826

Table 4: Optimal dynamic prices, expected demand and revenue for product 1 and product 2 in no capacity constraint condition

No capacity constraint has been taken into consideration while computing the optimal dynamic prices, demand and expected revenue above. Figure 10 and figure 11 below illustrate optimal dynamic pricing strategy and corresponding revenue for product 1 and product 2 in no capacity constraint condition.



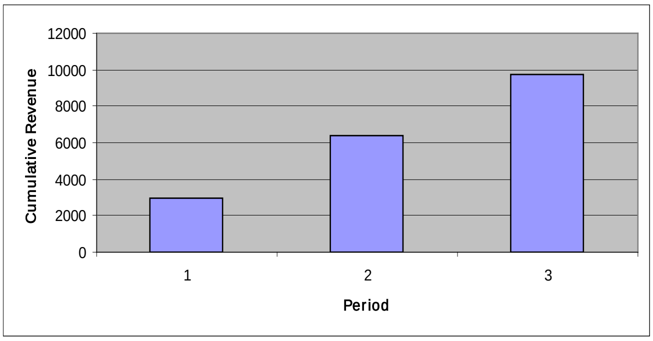
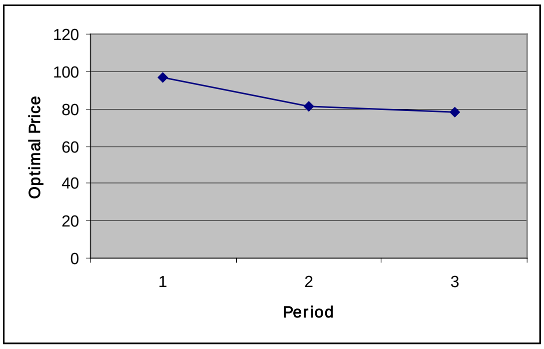


Figure 10: Optimal dynamic pricing strategy and corresponding revenues for product 1 in no capacity constraint condition.



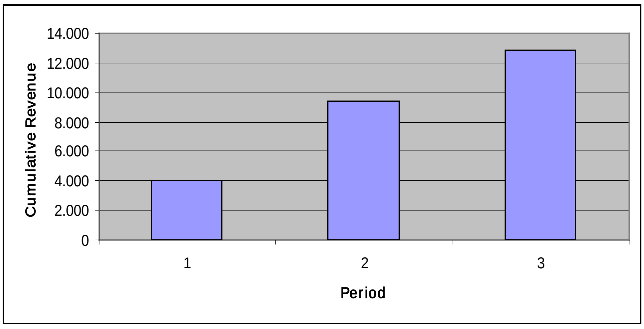


Figure 11: Optimal dynamic pricing strategy and corresponding revenues for product 2 in no capacity constraint condition

4.2 Computing The Optimal Dynamic Pricing Strategy for Limited Capacity Constraint Conditions

Another critical issue for the problem is the optimal price strategy change if capacity constraints are involved in the problem.

For example if the capacity is limited with 120 units for the product 2, then the optimal price strategy differs from the findings in the previous section. In this case, price based revenue functions between (49) and (51) have to be reformulated according to demand using the equations between (43) and (45).

<u>Demand based revenue functions for product 2</u>

Using equation (43), $d_1 = e^{4,746714 - 0,010304 \text{ x Price}}$, then,

Price = 1/0,010304 x [4,746714-ln (d_1)] is obtained, and, (52)

Revenue$_1$ = d_1/0,010304 x [4,746714-ln (d_1)] is found. (53)

Using equation (44), d_2 = -0,797 x Price + 129,9159, then,

Price = (129,9159-d_2) / 0,797 is obtained, and, (54)

Revenue$_2$ = (129,9159 x $d_2 - d_2^2$) / 0,797 is found. (55)

Using equation (45), d_3 = - 0,5625 x Price + 88,3121, then,

Price = (88,3121-d_3) / 0,5625 is obtained, and, (56)

Revenue$_3$ = (88,3121 x d_3-d_3^2) / 0,5625 is found. (57)

So, demand based revenue functions in (52), (54) and (56) for product 2 have been obtained by using the demand functions of the forecasting models which forecast better. These demand based revenue functions have been used as the objective function in the nonlinear programming model below.

List of Variables Used for The Non-linear Programming Model

d_1 = Demand corresponding to the optimal price used in period 1

d_2 = Demand corresponding to the optimal price used in period 2

d_3 = Demand corresponding to the optimal price used in period 3

λ = Lagrange Multiplier

C = Capacity Constraint (120)

Objective Function

Max d_1 /0,010304 x [4,746714-ln(d_1)] + (129,9159 x $d_2 - d_2^2$) / 0,797 + (88,3121 x d_3-d_3^2)/ 0,5625 (58)

Constraints

$$3,746714 / 0,010304 - \ln(d_1) / 0,010304 = \lambda \quad (59)$$

$$-2\, d_2 / 0,797 + 163,006 = \lambda \quad (60)$$

$$-2\, d_3 / 0{,}5625 + 156{,}999 \qquad = \lambda \qquad (61)$$

$$d_1 + d_2 + d_3 \qquad \leq 120 \qquad (62)$$

$$\lambda \text{ x } 120 - \lambda \text{ x } d_1 - \lambda \text{ x } d_2 - \lambda \text{ x } d_3 \qquad = 0 \qquad (63)$$

$$d_1 \qquad \geq 0 \qquad (64)$$

$$d_2 \qquad \geq 0 \qquad (65)$$

$$d_3 \qquad \geq 0 \qquad (66)$$

$$\lambda \qquad \geq 0 \qquad (67)$$

In the nonlinear programming model for product 2, equation (58) is the objective function.

Equations (59), (60) and (61) are marginal revenues and each one is the derivative of the revenue functions in the objective function.

For example,the derivative of $\frac{d_1}{0{,}010304} * [4{,}746714 - \ln(d_1)]$ is computed as in (59) by

$\frac{\partial(d_1 / 0{,}010304 * (4{,}746714 - \ln(d_1)))}{\partial(d_1)}$ =3,746714 / 0,010304 - ln (d_1) / 0,010304

Equation (62) indicates the limited capacity.
Equation (63) is called as complementary slackness. This means that if the limited capacity is not fully utilized, then the lagrange multipier must equal 0.
Equation (64), (65), (66) and (67) represent the positivity condition in the nonlinear programming model. To solve the nonlinear programming model, Lingo 12 software has been used. Figure 12 below shows the nonlinear programming model used in Lingo 12 in the limited capacity condition.

```
MAX=d1/0.010304*(4.746714-@log(d1)) + (129.9159*d2-@SQR(d2))/0.797 + (88.3121*d3-@SQR(d3))/0.5625;

3.746714/0.010304-@log(d1)/0.010304=λ;
-2*d2/0.797+163.006=λ;
-2*d3/0.5625+156.999=λ;
d1+d2+d3<=120;
λ*120-λ*d1-λ*d2-λ*d3=0;
d1>=0;
d2>=0;
d3>=0;
λ>=0;
```

Figure 12: Nonlinear programming model formulated by Lingo 12

In Figure 12, @log (d) function means ln(d) and it is the natural logarithm in base e. Besides, @SQR (d) function means the square of d. Solving the model above, demand in periods have is computed as follows:
d_1 = 31, d_2 = 53 and d_3 = 36. So, using equations (43), (44) and (45), optimal dynamic prices have been found out as follows:

Using equation (43), $d_1 = e^{4{,}746714 \text{ - } 0{,}010304 \text{ x Price}} = 31$

4,746714 – 0,010304 x Price = ln (31) = 3,433987

Price = 127,39 is obtained.

Using equation (44), d_2 = -0,797 x Price + 129,9159 = 53
Price = 96,50 is obtained by logarithm.

Using equation (45), d_3 = - 0,5625 x Price + 88,3121 = 36
Price = 93 is obtained by logarithm.

The corresponding revenue for the above price strategy is calculated as 12412 TRY. By the way, it has been observed that as the capacity for product 2 diminishes, optimal sales prices are in a tendency to rise. Figure 13 below shows the optimal dynamic pricing strategy in no capacity constraint condition and the optimal dynamic pricing strategy in 120 units of limited capacity condition for product 2.

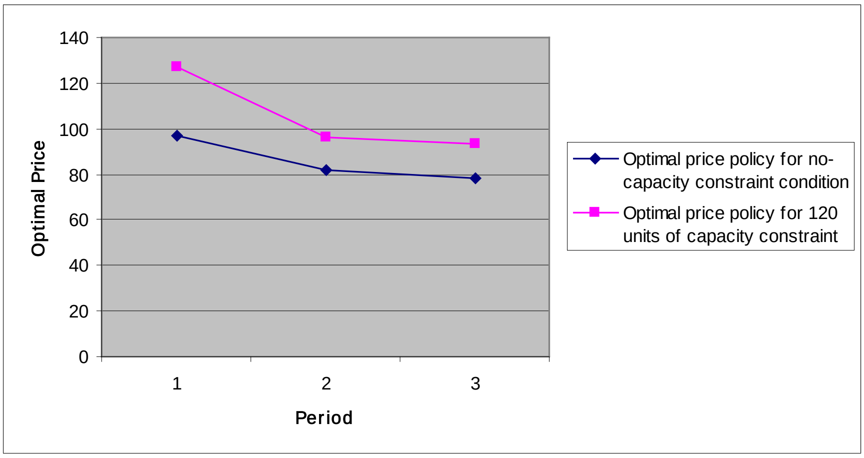


Figure 13: The optimal dynamic pricing strategy in no capacity constraint condition and the optimal dynamic pricing strategy in 120 units of limited capacity condition for product 2

4.3 Revising The Optimal Dynamic Pricing Strategy When Actual and Expected Sales Differ

In this case, using the suggested optimal price strategy, actual sales could be lower than the expected sales or alternatively actual sales could be higher than the expected sales. If actual sales are higher than the expected sales, the firm might not replenish inventory or might replenish inventory if possible.

4.3.1 Selling Less Than Expected Sales

Applying the optimal price strategy for product 2, after the first period, the on hand inventory at beginning of the second period should be 89 (120-31). However, if the company sells 20 units, the on hand inventory is 100 (120-20). So, the remaining price strategies until the end of the season changes. Lingo 12 model for this calculation given in Figure 14.

```
MAX=(129.9159*d2-@SQR(d2))/0.797 + (88.3121*d3-@SQR(d3))/0.5625;

-2*d2/0.797+163.006=λ;
-2*d3/0.5625+156.999=λ;
d2+d3<=100;
λ*100-λ*d2-λ*d3=0;
d2>=0;
d3>=0;
λ>=0;
```

Figure 14: The optimal dynamic pricing strategy for the remaining two periods for product 2 when actual sales are lower than expected sales

The optimal solution for the above model is d_2=60 and d_3=40. Using price based demand function for product 2, d_2 = -0,797 x Price + 129,9159 (44) and d_3 = - 0,5625 x Price + 88,3121 (45), p_2=87,72 and p_3=85,88 are found. So, the original optimal prices of 96,50 and 93 for period 2 and period 3 have decreased.

4.3.2 Selling More Than Expected Sales

Applying the optimal price strategy for product 2, after the first period, the on hand inventory at beginning of the second period should be 89 (120-31). However, if the company sells 50 units, the on hand inventory is 70 (120-50). In this case, the firm might not replenish inventory or might replenish inventory if possible.

a) Selling More Than Expected and No Replenishment

Lingo model for this case is given in Figure 15.

```
MAX=(129.9159*d2-@SQR(d2))/0.797 + (88.3121*d3-@SQR(d3))/0.5625;

-2*d2/0.797+163.006=λ;
-2*d3/0.5625+156.999=λ;
d2+d3<=70;
λ*70-λ*d2-λ*d3=0;
d2>=0;
d3>=0;
λ>=0;
```

Figure 15: The optimal dynamic pricing strategy for the remaining two periods for product 2 when actual sales are more than expected sales and no replenishment

The optimal solution for the above model is d_2=42 and d_3=28. Using d_2 = -0,797 x Price + 129,9159 (44) and d_3 = - 0,5625 x Price + 88,3121 (45), p_2=110,31 and p_3=107,22 are found. So, the original optimal prices of 96,50 and 93 for period 2 and period 3 have increased. By the way, the revenue function value equals to 7635.

b) Selling More Than Expected and Inventory Is Replenished

If ζ is the deviation between actual sales and expected sales at the first period, and if the firm is able to replenish inventory by ζ,, then ζ is 19 (50-31) and the following model which is given in Figure 16 can be performed.

```
MAX=(129.9159*d2-@SQR(d2))/0.797 + (88.3121*d3-@SQR(d3))/0.5625;

-2*d2/0.797+163.006=λ;
-2*d3/0.5625+156.999=λ;
d2+d3<=70+ζ;
λ*(70+ζ)-λ *d2-λ*d3=0;
d2>=0;
d3>=0;
λ>=0;
ζ=19;
```

Figure 16: The optimal dynamic pricing strategy for the remaining two periods for product 2 when actual sales are more than expected sales and inventory is replenished

The optimal solution for the above model is d_2=53 and d_3=36. Using d_2 = -0,797 x Price + 129,9159 (44) and d_3 = - 0,5625 x Price + 88,3121 (45), p_2=96,50 and p_3=93 are found. By the way, the revenue function value equals to 8463 which is 828 more than 7635. So, we can conclude that the firm had better replenish inventory if possible to gain more revenue if it sells more than expected.

5. RESULTS AND DISCUSSIONS

In a dynamic pricing application, a firm with a limited capacity tries to answer the question of which prices to use in time is necessary in order to get the maximum revenue. Making this decision, probably the most critical issue is to find out how demand occurs. So, in this study, firstly it has been tried to find out by which forecasting model the relevant demand could be expressed well. For this reason, RRMSE measures of the forecasting models have been compared. By this way, the forecasting model which forecast better have been selected. Since the data used in the study is count data, appropriate forecasting model has been tried to select among poisson regression and support vector machine models. In most cases, poisson regression has given better results than support vector machine. So, it can be suggested that researchers should heavily depend on classical count data models in cases which the number of data is small. However, the researchers might still take advantage of other models by comparing count models with others fitting count data.

According to the findings, for unlimited capacity condition, it has been found that the optimal sales prices should fall as the the selling period comes to an end. It has been also found that the rate of the decreases in the sales price should change from time to time. For example, the first decrease in price for product 2 has been computed as 16 % whereas the second decrease has just been computed as 3,7 % . Another point observed has been that the decrease in optimal prices should change from product to product. For example, the first decrease in product 1 has been computed as 35,3 % while 16 % in product 2.

Another important point is that any policy different from the optimal dynamic policy will always bring less revenue than the optimal one. For example, the revenue of any other pricing policy for product 2 would be less than 12826 TRY. In addition to the findings above, limited capacity condition has also been analyzed by nonlinear programming model and new optimal dynamic prices for this condition have been found. The results have indicated that optimal prices should fall as the season comes to end. It has also been discovered that the decreases in price might change from period to period. For example, the first decrease in price for product 2 in 120 units of limited capacity has been computed as 24 % whereas the second decrease has just been computed as 3,6 % .

In the study, the difference between the optimal dynamic pricing strategies of unlimited and limited capacity conditions has been compared too. As a result, optimal prices for limited capacity condition have been found higher than unlimited capacity condition. For example, optimal dynamic prices for product 2 in 120 units of limited capacity condition have been found 127,39, 96,50 and 93 TRY respectively. On the other hand, in unlimited capacity condition, optimal prices for product 2 have been computed as 97,04, 81,50 and 78,50 TRY respectively.

By the way, according to the findings, the revenue obtained in unlimited capacity condition has been found higher than the revenue in limited capacity condition. For example, the revenue for product 2 in unlimited capacity condition has been found as 12826 TRY. Whereas, it has been computed as 12412 TRY in 120 units of limited capacity condition.

The findings above is based on the data used in the study. However, using different data and the same procedure, one may get naturally a different pricing policy such as first a falldown and then a steady price afterwards but the proposed methodology remains the same. In conclusion, the solution and optimization approach described and implemented in this paper can be extended to other and similar real world-pricing decisions in retailing.

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